MAS 720: Probabilistic Methods in Algorithm Design and Analysis

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"It is remarkable that this science, which originated in the consideration of games and chances, should have become the most important object of human knowledge... The most important questions of life are, for the most part, really only problems of probability"

— Pierre Simons, Marquis de Laplace (1749–1827).

Randomization and Computing

"If somebody would ask me, what in the last 10 years, what was the most important change in the study of algorithms I would have to say that people getting really familiar with **randomized algorithms** *had to be the* **winner**.*"*

— Donald Knuth ("Randomization and Religion", 1999).

"In some very real sense, computation is inherently randomized. It can be argued that the probability that a computer will be destroyed by a meteorite during any given microsecond of it operation is at least 2^{-100}*".*

— *Christos Papadimitriou.*

Probability and Computing

- Randomized algorithms - random steps help!
- Probabilistic analysis of algorithms - Why hard problems are sometimes easy to solve in practice?
- Probabilistic deduction, statistical inference, machine learning...

Applications: Communication networks; Cryptography; Search engines; Fast data structures; Scheduling; Optimization algorithms; Simulation and Modeling, AI reasoning; Learning; Bioinformatics, Quantum Computing, Complexity Theory ...

Topics

Theme: Emphasis on **Probabilistic techniques** — illustrated by **various algorithmic applications**.

1. Introduction to probability theory

— Expectation, Linearity of Expectation. — Moments, Variance. — Deviation bounds: Markov, Chebyshev, and Chernoff-Hoeffding Bounds.

2. Introduction to Randomized Algorithms and Probabilistic Analysis

— Las Vegas and Monte Carlo Algorithms. — Karger's Min Cut Algorithm. — Fingerprinting technique, Pattern Matching. — Randomized Quicksort. — Randomized Selection. — Stable Marriage Problem. — Packet Routing. — Linear time Minimum Spanning Tree Algorithm. — Shortest Paths. — Skip Lists

3. Balls and Bins Paradigm

— Birthday Paradox. — Coupon Collector's Problem. — Poisson Approximation. — Hashing, applications. — Two choices paradigm.

4. Chernoff Bounds Under Dependent Settings — Negative Dependence — Local Dependence — Janson's inequality

5. Further Techniques for Tail Bounds and Applications

— Martingales — Azuma's Inequality — Method of Bounded Differences — Applications

6. Random walks and Markov chains

— Markov Chain Basics. — 2-SAT and 3-SAT algorithms. — Random Walks on Graphs and Undirected Connectivity. — Variation Distance and Mixing Time. — Expander Graphs. — Coupling and Convergence. — Algorithmic Applications.

7. The Probabilistic Method.

— Counting and Expectation techniques, Applications. — Second Moment Method. — Lovasz

Local Lemma. — Algorithmic applications of the Lovasz Local Lemma.

8. Distributed Network Algorithms.

— Maximal Independent Set (Luby's algorithm). — Distributed Dominating Set Problem. — Byzantine Agreement.

9. Approximation Algorithms

— Approximate Counting — Randomized Rounding — Probabilistic Tree Embeddings

10. Random Graphs.

— Models of Random Graphs. — Threshold Phenomena. — Connectivity and Giant Component. — Random Graph Algorithms.

References

- Probability and Computing by M. Mitzenmacher and E. Upfal.
- Concentration of Measure for the Analysis of Randomized Algorithms by Dubhashi and Panconesi.
- Randomized Algorithms by R. Motwani and P. Raghavan.
- Probabilistic Method by Alon and Spencer.
- A First Course in Probability by S. Ross (useful reference for probability).

Grading

Homeworks (30%):

Biweekly assignments.

Individually written (preferably in Latex).

Work **must** be submitted on time.

Research Project (20%).

Each project will consist of a team of one or (preferably) two students.

Each team will work on an assigned (theoretical) problem.

Each team has to submit a project report on their work.

Final Exam (50%).

Academic Dishonesty policy: All submitted work should be on your own. Copying or using other people's work (including from the Web) without proper reference is prohibited. It will result in $-MAX$ points, where MAX is the maximum possible number of points for that assignment. Repeat offense will result in getting a failure grade in the course and reporting to the Chair's office.